SEVENTH SUMMER MEETING OF THE MATHEMATICAL ASSOCIATION OF AMERICA.

By invitation of the University of Rochester, the seventh summer meeting of the Association was held at this University on Wednesday and Thursday, September 6–7, 1922, in conjunction with, and immediately preceding, the summer meeting of The American Mathematical Society. There were 104 in attendance at the sessions, including the following 75 members of the Association:

N. H. ANNING, University of Michigan.

R. C. ARCHIBALD, Brown University.

FLORENCE L. BECKER, High School, Webster, N. Y.

W. J. BERRY, Brooklyn Polytechnic Institute.

WILLIAM BETZ, East High School, Rochester.

R. L. BORGER, Ohio University.

W. G. BULLARD, Syracuse University.

R. W. BURGESS, Brown University.

W. D. CAIRNS, Oberlin College.

W. B. CARVER, Cornell University.

E. H. CLARKE, Hiram College.

H. E. COBB, Lewis Institute.

A. B. COBLE, University of Illinois.

C. E. COMSTOCK, Bradley Polytechnic Institute.

LENNIE P. COPELAND, Wellesley College.

A. R. CRATHORNE, University of Illinois.

F. F. DECKER, Syracuse University.

H. A. DoBELL, Colgate University.

L. P. EISENHART, Princeton University.

H. J. ETTLINGER, University of Texas.

B. F. FINKEL, Drury College.

C. A. FISCHER, Trinity College.

W. B. FORD, University of Michigan.

R. E. GAINES, Richmond College.

A. S. GALE, University of Rochester.

R. E. GILMAN, Brown University.

C. F. GUMMER, Queen's University.

C. E. HARRINGTON, University of Buffalo.

E. R. HEDRICK, University of Missouri.

H. C. HICKS, Brown University.

T. H. HILDEBRANDT, University of Michigan.

W. A. HURWITZ, Cornell University.

NELLE L. INGELS, Interstate Commerce Commission.

L. C. KARPINSKI, University of Michigan.

O. D. KELLOGG, Harvard University.

H. W. KUHN, Ohio State University.

W. D. LAMBERT, U. S. Coast Survey.

SOLOMON LEFSCHETZ, University of Kansas.

N. J. LENNES, University of Montana.

ANNA D. LEWIS, Lake Erie College.

T. R. LONG, University of Rochester.

JOHN MATHESON, Queen's University.

G. A. MILLER, University of Illinois.

NORMAN MILLER, Queen's University.

G. R. MIRICK, East High School, Rochester.

C. N. MOORE, University of Cincinnati.

C. C. MORRIS, Ohio State University.

F. W. OWENS, Cornell University.

Mrs. F. W. OWENS, Cornell University.

L. R. PERKINS, Middlebury College.

A. D. PITCHER, Western Reserve University.

L. C. PLANT, Michigan Agricultural College.

V. E. POUND, University of Buffalo.

S. E. RASOR, Ohio State University.

F. W. REED, Cornell University.

L. W. REID, Haverford College.

HARRIS RICE, Worcester Polytechnic Institute.

R. G. D. RICHARDSON, Brown University.

E. D. ROE, Syracuse University.

HAZEL E. SCHOONMAKER, Western College.

W. H. SHERK, University of Buffalo.

W. G. SIMON, Western Reserve University.

H. E. SLAUGHT, University of Chicago.

CLARA E. SMITH, Wellesley College.

VIRGIL SNYDER, Cornell University.

K. D. SWARTZEL, University of Pittsburgh.

W. A. TITSWORTH, Alfred College.

B. L. WAITS, Florida A. and M. College.

C. W. WATKEYS, University of Rochester.

J. H. WEAVER, Ohio State University.

J. H. M. WEDDERBURN, Princeton University.

D. E. WHITFORD, University of Rochester.

F. B. WILLIAMS, Clark University.

C. H. YEATON, Oberlin College.

J. W. YOUNG, Dartmouth College.

From this list it will be noticed that a very good representation was afforded from the different regions covered by the Association. The joint dinner of the two organizations occurred on Thursday evening with Professor Eisenhart as presiding officer. Professor Archibald, the president of the Association, gave by

invitation an extended and informing account of various phases of his trip through Europe during the past spring and summer, and Professor Snyder gave an interesting description of the exercises at the seven hundredth anniversary of the founding of the University of Padua. Some extracts from Professor Archibald's address will be printed in one of the forthcoming issues of the MONTHLY. A hearty vote of thanks was given to the Board of Governors of the Oak Hill Country Club for their courtesy in extending the facilities for this dinner to the two organizations.

There was a notable variety of events in connection with the meetings at Rochester. On Wednesday afternoon at the close of the program, a short chamber concert was given for the members of the Association and the Society in Kilbourn Hall of the new Eastman School of Music, at which a program consisting of violin, piano, and vocal selections was given by the advanced students of the School of Music, and by Mr. Gleason, the head of the Organ Department. The visitors took advantage of the free evenings to attend the performances at the Eastman Theatre, which had been opened on Monday of that week; this theatre combines in notable fashion programs of motion pictures and of music. The tour of inspection through the Research Laboratory of the Eastman Kodak Company is referred to later. On Thursday afternoon, at the close of the Society program, the visiting members were taken in automobiles on a general tour of inspection of Rochester and its surroundings, particularly the Durand-Eastman Park near Lake Ontario, and the grounds of the Oak Hill Country Club where the joint dinner was held on Thursday evening. On Friday afternoon the opportunity was given for an inspection of the plant of the Bausch & Lomb Optical Company. An unusual amount of attention was paid to the mathematical meetings by the daily newspapers of Rochester which each day reported quite fully the details of the programs.

A considerable number of men were housed comfortably in Kendrick Hall, the other members of the Association staying at the hotels which were not distant from the University. Many provisions were made to enhance the comfort and pleasure of the visiting guests, and the Association recognized these courtesies in a formal vote of thanks which was offered at the meeting on Thursday morning, with particular mention of Professor Gale and his associates in the department of mathematics, the Eastman School of Music, the Eastman Kodak Company, and the Bausch & Lomb Company.

The following papers were given; abstracts of these papers follow, the numbers corresponding to the numbers in the lists of titles.

SESSION OF THE ASSOCIATION ON THE PRESENT STATUS OF UNIFIED MATHEMATICS.

(1) "The problem of organizing freshman college courses" by Professor J. W. YOUNG, Dartmouth College.

(2) "Historic consideration of unified mathematics" by Professor L. C. KARPINSKI, University of Michigan.

(3) "Some aspects of unified mathematics for freshmen" by Professor R. W. BURGESS, Brown University.

(4) "Internal reasons for unification" by Professor C. E. COMSTOCK, Brooklyn Polytechnic Institute.

(5) General discussion, led by Professors F. B. WILLIAMS, Clark University, K. D. SWARTZEL, University of Pittsburgh, and C. H. YEATON, Oberlin College.

1. The gist of Professor Young's paper will appear in an early issue of the MONTHLY.

2. The purpose of Professor Karpinski's paper was to show that the greatest teachers of the world's history employed methods, materials, and illustrations to enrich their instruction, as far as such material was available. Even such a text as Euclid's *Elements* contains, in addition to the seventh to tenth books which are strictly theoretical arithmetic, a wealth of algebraic material. Similarly Archimedes in his *Sphere and Cylinder* solves by the intersection of conics a cubic equation and gives what corresponds to analytical conditions for the existence of real roots of a given cubic. Ptolemy, the great geographer-astronomer, introduces into his astronomical treatise whole sections on geometry and on the computation of chords. The trigonometry in Ptolemy's *Almagest* is developed as completely as it would be in a treatise devoted strictly to mathematics rather than to astronomy. With the great Arabic text-book writers and scientists, with Al Khowarizmi and Omar al Khayyam, in particular, the treatises on algebra were enlivened and enriched by a mass of geometrical and practical illustrations. The same was true with Régiomontanus and with such writers as Euler and Newton. Our texts, prepared for a more immature audience, have made the great error of insistence upon strictly logical unity, rather than upon psychological adaptation to the readers for whom the texts are prepared. Historically, at least, good authority can be found for a broader treatment on a unified basis.

3. Taking as the key principle of unified mathematics for college freshmen the statement that subject matter and methods of instruction should be determined by the present and future needs of students who take no more mathematics, Professor Burgess urged that a searching examination of the various activities of life must be made to bring out just what mathematical attainments are really needed. Casual and uncritical statements, such as that of the graduate who says he never uses mathematics, are not to be accepted as evidence without scrutiny. Professor Burgess then suggested a division of the uses of mathematics into three main classes,—purely mathematical or geometrical, experimental, and statistical; and urged that increased attention be paid to the last type as the one which is most frequently needed subsequently by students. He urged that mathematicians recognize statistical mathematics as the mathematics of aggregates in which the individual elements differ from each other to a significant extent, and that they look upon even elementary statistical methods as worthy of serious attention. The statistical topics suggested for the freshman course are percentage analysis, frequency distributions, simple averages and diagrams, index numbers, and a rough determination of trend lines. Except

for the introduction of the statistical point of view, he joined with others in regarding the function concept as a unifying principle. In regard to the methods to be used, he suggested that extreme emphasis on securing expert facility is unwise, that proofs are of secondary importance in this course, that the needs of the individual class should modify the conduct of the course, and that problems should be selected from a wide field.

4. Professor Comstock discussed the four subjects, college algebra, trigonometry, analytic geometry and calculus. The central ideas of these subjects may be stated as follows: college algebra, a complex of somewhat unrelated topics, such as the solution of equations and the transformation of expressions containing the common functions of algebra; trigonometry, the algebra of the circular functions and the arithmetic of the triangle; analytic geometry, characterized by its three problems, finding the locus of an equation, finding the equation of a locus, and finding the properties of conics by algebraic methods; and calculus, concerned with the changes of functions due to changes in the independent variables. Now there has steadily developed an interpenetration of the fields which surround these central ideas, and this may be considered in two aspects: the more advanced subjects must build upon material from the lower courses; certain topics and methods of one subject are found helpful in presenting the concepts of another subject. For example, calculus calls on geometry for illustration; and geometrical concepts are found exceedingly helpful in presenting analytic processes; also the graph has invaded algebra and trigonometry. In fact, a rudimentary unification of all these subjects is actually going on.

The combination of elementary algebra and plane geometry in one course is not desirable, since they are two cultures made up of two sets of distinctive habits which the student must acquire; the exposure to each must be long enough and sufficiently free from interruption for the ideas to set firm as distinct habits. College students, however, are more mature and have these two lines of thought fixed. In separate college courses much time is lost on dislocated topics and unnecessary methods. Partial fractions should be left until they are needed in integration. A more adequate treatment of series can be given when calculus has been acquired. The introduction of the derivative in analytic geometry improves the treatment of many topics, notably the determination of tangents, the plotting of curves and the study of their properties. The student should be introduced to the most fruitful concepts of mathematics as early as possible. There should be a whole-hearted willingness to use any idea or method that will be most helpful at the time, whether the order is traditional or not.

The function concept runs through all four of the subjects that we have considered and may well be used as a unifying bond, the more so because of its three modes of expression, arithmetical, algebraic and geometric. Sufficient time must be given each topic to afford a deep infusion, and the connection between topics should be made evident. A two-year course is desirable, but students who take one year will have acquaintance with the most valuable methods.

5. Professor Williams said that the whole course should be developed in a

logical way, that it is necessary in all colleges to give the students some review of their previous material. It is not so much a question of time; we need to take them where, on the whole, they are found, and give them as much as can be done during the year. He outlined the course which he has given each year since 1907, a course comprising such topics as: the location of points by coördinates, trigonometric functions and applications, motion of a point in a plane, the intersection of two lines, the different approaches to determinants, the motion of three points on a line, and three lines through a point, with a suggestion of the principle of duality, tangents, limits, introduction to calculus, including its first applications. There should be proper attention paid to symbolic representation so as to avoid confusion in the student's mind on this score. The result of his experience of fifteen years is that this kind of a course is not merely possible but is very valuable.

Professor Swartzel believes that it is possible to write a satisfactory text in uniform mathematics for freshmen, but that this task is still to be done. The teaching of functions in this way does undoubtedly give the students a meaning of the term, but not the general meaning of "function" which we desire, such as is done in the traditional course. There should be a further discussion not merely of the topics which have been dropped in making up this course, but also of the educational side of the question,—a development of the mind toward clear, accurate thinking. The new book for this course should pay attention to the best development of the mind and the incorporating of the new relations which may well be added to the course for freshmen. Professor Swartzel has found it difficult in a course in uniform mathematics to get a sort of examination that shall be satisfactory to the teacher and to the pupils, examinations that will readily and fairly test the student's mastery. Further it is difficult to make the subject-matter coherent, and there is a lack of desirable drill in this course. Do not these difficulties make it evident that we are trying to do too much in a one-year course? He feels that the new style of course is best suited to the mediocre students, who at least appear to do a little better under this plan. A great opportunity lies at hand in the line of segregation of students according to their ability; this will incidentally help in this trial of uniform mathematics.

Professor Yeaton has found that much of the success of the uniform course depends on the enthusiasm of the particular teacher giving the course. He personally feels that this course, including the applications of calculus, is good for students who are sure that they will not continue after the first year, and that it arouses their interest and imagination. The course, however, has the fault of including too many topics, a great difficulty unless we plan a two-year course in its entirety. For the good students it seems quite unfortunate to crowd out the analytic geometry of the first year inclusive of some good serious study of conic sections and the necessary drill in such work.

Mr. Lambert of the Coast Survey raised the question whether the free combination of mathematical methods by the ancients was meant for students of the same grades as those in our colleges. He has found in setting examination ques-

tions for government examinations that questions of a mixed sort have been too difficult and that he had to resort to questions of the traditional sort.

Professor Lennes has seen no way of getting at the questions except by actual tests, and he almost despairs of a result for the reason that one's predisposition toward, say, uniform mathematics, as well as other lines of work, will warp one's judgment even with the best of intentions to be unprejudiced. He has found it hard to get the reaction of the pupil to the three or four pills separately administered in comparison with all given together in one dose.

Professor Plant has felt in opposition to Professor Lennes's point of view that we are getting back to what was the original method of one pill, not four pills in one, and that in his observation the use of various of the present-day texts in uniform mathematics has undoubtedly brought about a satisfactory mental development in the pupils.

Professor Berry has found that a difficulty arises in the case of men training for a special purpose who hold mathematics as a tool, and he concludes that for these the proper course in uniform mathematics has not yet appeared. He has used with success a sort of infiltration process, transferring topics from the traditional order to the point in a course where these are needed.

Second Session of the Association.

(6) "Contradictions in the literature of the group theory," retiring presidential address by Professor G. A. MILLER, University of Illinois.

(7) "An English text of mathematics written about 1810" by Professor ELIZABETH B. COWLEY, Vassar College.

(8) "Impressions of mathematics and mathematical instruction in Italian universities" by Professor VIRGIL SNYDER, Cornell University.

6. The address of the retiring president, Professor Miller, will shortly appear in this MONTHLY.

7. Miss Cowley's paper concerned a three-volume mathematical work in manuscript and will be published in a later issue of this MONTHLY.

8. Professor Snyder described mathematical instruction in Italy. In the preparatory course of twelve years in Italian schools, the range and scope are fairly similar to our own, but the pupils are better trained than ours in mathematics and the languages. In the university, the possible range of studies for a beginner is large, but after the initial selection, the subsequent range is very limited. Those selecting pure mathematics, for example, may have one course in chemistry, or in experimental physics, but with this exception, the entire college course is spent on pure mathematics. During the first year the average student takes fifteen hours, in the second he takes twelve, and in the third and fourth nine each and a thesis. If the term examinations are all passed, and the thesis accepted, and defended in a brief formal examination, the student graduates with the degree of doctor of mathematics. All instruction is by the formal lecture method, frequently not accompanied by any exercise period; there is no

control over attendance; a brief oral examination (twenty minutes) at the end of the year is the only criterion upon which to base the estimate of the quality of the year's work. Projective geometry is begun the first year, and the student starts analytic geometry at the same time; both continue throughout the first year. The calculus is not begun until the second year. Everywhere great emphasis is put on reasoning, and the vehicle most frequently and extensively employed for this purpose is synthetic geometry, in which the students become remarkably skillful. While the Italians have contributed essentially to the development of almost all general branches of mathematics, the present theory of algebraic geometry is preëminently an Italian product, and is perhaps the most important Italian contribution to modern mathematics.

SESSION AT THE RESEARCH LABORATORY OF THE EASTMAN KODAK COMPANY.

(9) "The physical problems involved in photographic research" by Mr. L. A. JONES, of the Research Laboratory.

(10) "Mathematics puzzles as an introduction to investigation" by Professor W. B. CARVER, Cornell University.

(11) "The present status of the formal discipline controversy" by Professor N. J. LENNES, University of Montana.

(12) Inspection of the Research Laboratory.

9. Mr. Jones described the general nature of the problems treated in the Research Laboratory, covering such topics as the following: the amount of energy to be used in producing the photo-chemical effect, the instability of the nitrocellulose base, the use of dyes in making colored lights, sensitizing agents, etc. As an example of the sort of research carried on in the Laboratory, he showed by graphical representation the nature of the study of the sensitiveness of the ordinary portrait photographic plate, the bichromatic and the panchromatic plates. He made evident, in what at once appeared to those present as a functional dependence, the various factors which determine how perfectly the photographic process reproduces the impression gained by the direct vision of an object.

10. The paper by Professor Carver will appear in a later issue of this MONTHLY.

11. This paper contained a resumé and an evaluation of the arguments and experimental investigations which were supposed to have exploded the doctrine of formal discipline, a doctrine which for practical purposes may be said to deal with the effect of training in one ability upon other abilities. Professor Lennes pointed out that the doctrine of the "faculties of the mind" which was used earlier as the basis for the doctrine of formal discipline is not necessary for the latter. Moreover, the experimental investigations that have been made deal only with peripheral activities in which reflection and judgment have little or no part. Finally it is pointed out that the very simple (necessarily so) experiments that have been made show a considerable carrying over into other activities of the effects of training. Hence it is concluded that for practical purposes the

older doctrine of formal discipline so far from being exploded has not even been disturbed.   It does require and is in fact in the process of receiving restatement to make it conform to the newer psychology, but its essential content need not be greatly different from the old doctrine.

12. Following the Thursday morning program and business meeting, members of the Association and other visitors were conducted by various members of the research staff through the Research Laboratory, where they were given some notion of the manifold activities involved in this side of the work of the Eastman Kodak Company's business.   Costly apparatus of the most elaborate sort, a large range of chemical, optical and other physical equipment, fills the rooms in what seems to the tyro a crowded manner, but what the Laboratory's results show to be a systematic working plan.   Many contacts of interest were made by the visitors on this tour of inspection which was finished all too quickly.

MEETING OF THE TRUSTEES OF THE ASSOCIATION.

The following 26 persons and one institution, on applications duly certified, were elected to membership.

To individual membership.

P. L. ARMSTRONG, A.M. (Southwestern Presb. Univ.).   Instr., Georgia School of Tech., Atlanta Ga.

Sister M. BERNADITA, A. B. (Univ. of Colorado).   Head of dept. of math., Loretto Heights Coll., Loretto, Colo.

W. H. BOERCKEL, A.B. (Pennsylvania).   Instr., Georgia School of Tech., Atlanta, Ga.

R. W. BRINK, Ph.D. (Harvard).   Asso. prof., Univ. of Minnesota, Minneapolis, Minn.

A. H. COWLING, A.M. (Texas).   Prof., East Texas State Normal Coll., Commerce, Tex.

ELIZABETH S. DICE, A.B. (Texas).   Teacher, High School, Dallas, Tex.

H. A. DoBELL, A.B. (Syracuse).   Instr., Colgate Univ., Hamilton, N. Y.

J. L. DRISCOLL, A.B. (Washington and Lee).   Instr., Georgia School of Tech., Atlanta, Ga.

E. F. FREIER, A.B. (Minnesota).   Eveleth, Minn.

J. C. HINTON, A.M. (Georgia).   Dean, Wesleyan Coll., Macon, Ga.

D. N. KINGERY, A.M.   Prof., Macalester Coll., St. Paul, Minn.

Sister LAURENTINE MARIE, A.B. (Trinity Coll., Washington).   Instr., Emmanuel Coll., Boston, Mass.

O. B. LOEWEN, A.B. (Bethel Coll.).   Head of dept. of math., Ottawa Univ., Ottawa, Kans.

T. R. LONG, A.B. (Rochester).   Asst., Univ. of Rochester, Rochester, N. Y.

R. L. NEWLIN, B.S. (Guilford).   Instr., Guilford Coll., Guilford College, N. C.

R. L. O'QUINN, A.B. (Louisiana State).   Instr., Louisiana State Univ., Baton Rouge, La.

ELSIE M. PLAPP, M.S. (Chicago).   Asst. prof., Hollins Coll., Hollins, Va.

V. E. POUND, Ph.D. (Toronto).   Instr., Univ. of Buffalo, Buffalo, N. Y.

R. S. SHEPPARD, M.A. (Alberta).   Prin., High School, Strathcona, Alb., Canada.

G. S. SMITH, A.M. (Texas).   Head of dept. of math., High School, Port Arthur, Tex.

VIRGIL SNYDER, Ph.D. (Göttingen).   Prof., Cornell Univ., Ithaca, N. Y.

G. T. TRAWICK, B.S. in E.E. (Georgia School of Tech.).   Instr., Georgia School of Tech., Atlanta, Ga.

ANN VAN OEL, B.S. (Drake Univ.).   Teacher, High School, Greene, Ia.

V. H. WELLS, Ph.D. (Michigan).   Prof., Carleton Coll., Northfield, Minn.

R. I. WHITE, A.B. (Pennsylvania).   Instr., Georgia School of Tech., Atlanta, Ga.

E. B. WILSON, B.S. (Chicago).   Instr., Georgia School of Tech., Atlanta, Ga.

To institutional membership.

PROVIDENCE COLLEGE, Providence, R. I.   Pres. William Noon, Official representative.

These names, together with those of all others elected to membership during the past three years, have been printed in the *Register of Officers and Members*, 1922.

The Trustees voted to recommend to the business meeting of the Association the draft of the proposed By-Laws which was presented on Thursday morning.

The resignation of Professor A. A. Bennett as Editor-in-Chief of the MONTHLY was received and accepted, it being understood that his duties terminate at the close of the volume for 1922.   Professor W. B. FORD of the University of Michigan was appointed Editor-in-Chief beginning with the issue for January, 1923.   Some of the Associate Editors for the year 1923 were appointed, the full list to be completed at the December meeting.   Professor C. H. YEATON was appointed Assistant Secretary of the Association.   Professor L. C. KARPINSKI was appointed Librarian of the Association, and Professor MARY EMILY SINCLAIR, Assistant Librarian.

The Trustees approved the formation of a Southeastern Section to include Alabama, Florida, Georgia, North Carolina and South Carolina, and possibly Tennessee.

The Trustees also transacted various matters of business concerning the Library of the Association, and the extension of the Association's subvention to the ANNALS.

The President was requested to send the greetings of the Association to Professor Neuberg of the University of Liége, and to the Belgian Mathematical Society whose official organ is *Mathesis*.

BUSINESS MEETING OF THE ASSOCIATION.

A draft of the proposed By-Laws of the Association, which had been considered at great length by the Trustees, was recommended by them to the Association; because of the delay of five months in the printing of the MONTHLY, printed notice of the proposed amendments and of the change in the Articles of Association was sent to the members of the Association one month before the summer meeting, the purpose being to comply with the spirit of the requirement of the By-Laws to give such notice through the official journal.   The By-Laws as amended were unanimously adopted at the meeting on Thursday morning, September 7th, 1922, as well as the modifications of the Articles of Association which changed the number of Trustees from nineteen to twenty so as to include a Librarian.[1]

W. D. CAIRNS, *Secretary-Treasurer.*

[1] These By-Laws were printed with the *Register*, 1922, 44-46.